Sparse ℓ_1 Regularisation of Matrix Valued Models for Acoustic Source Characterisation

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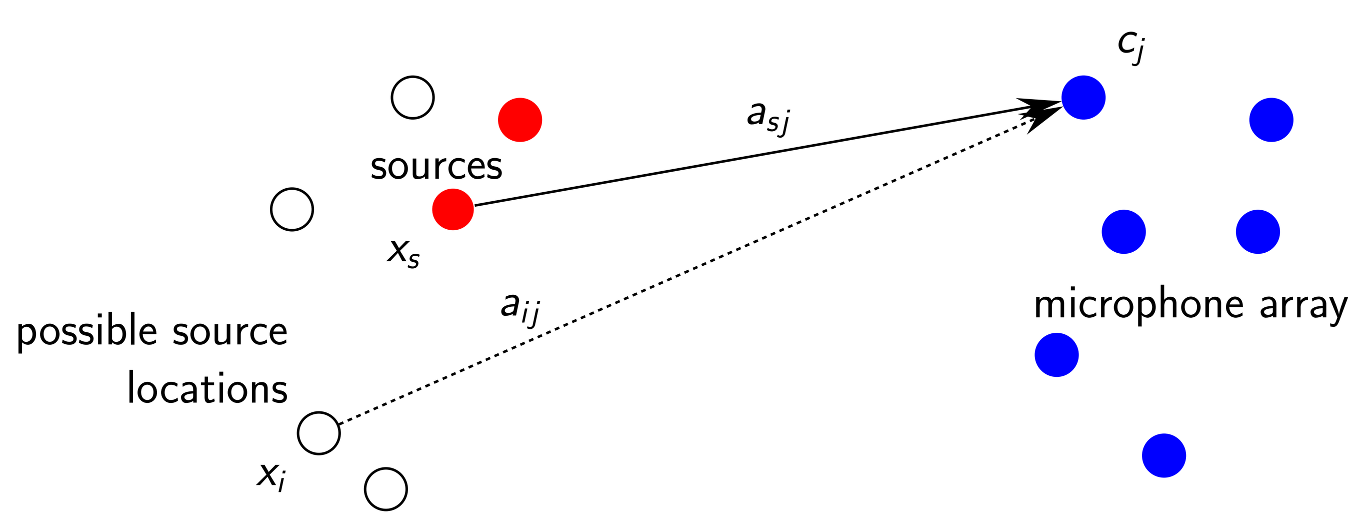
Acoustic Source Characterisation

Experimental Setup:

- n microphones at known locations
- m known and fixed possible source locations x_i
- $s \ll m$ unknown sound sources (monopoles)
- sound sources are uncorrelated
- emitted sound is recorded at all microphones

Task:

- locate all sound emitting sources



Physical Model:

- sound pressure recorded at microphone j:

$$c_j := \sum_i \underbrace{\frac{r_{0,j}}{r_{i,j}} \exp\left(\imath\omega \frac{r_{0,j} - r_{i,j}}{c_0}\right)}_{=:a_{ij}} x_i$$

- using the cross-spectral matrix $C := \mathrm{E}[cc^\top]$:

$$C = A \underbrace{\mathrm{E}[xx^\top]}_{=:X} A^\top$$

Task:

- find diagonal matrix X with sparse diagonal

Our Contributions

- apply popular method from imaging to engineering task
- combine matrix differential calculus with fast numerics in $\mathbb{C}$
- new model for the task at hand

Our Model

We seek a diagonal matrix $X \in \mathbb{C}^{m,m}$ with sparse diagonal that verifies

$$AXA^\top = C, \quad A \in \mathbb{C}^{n,m},\ C \in \mathbb{C}^{n,n}$$

We solve either

$$\operatorname*{arg\,min}_{X \in \mathbb{C}^{m,m}} \left\{ \frac{1}{2} \|AXA^\top - C\|_F^2 + \lambda \|X\|_1 \right\} \tag{1}$$

$$\operatorname*{arg\,min}_{X \in \mathbb{C}^{m,m}} \left\{ \frac{1}{2} \|AXA^\top - C\|_F^2 + \|W \circ X\|_1 \right\} \tag{2}$$

$$\begin{cases} \operatorname*{arg\,min}_{X \in \mathbb{C}^{m,m}} \left\{ \frac{1}{2} \|AXA^\top - C\|_F^2 + \lambda \|X\|_1 \right\} \\ \text{under the constraint that } X \text{ is a diagonal matrix} \end{cases} \tag{3}$$

with parameters $\lambda \in \mathbb{R}$, $W \in \mathbb{R}^{m,m}$

A Solving Strategy Based on Split Bregman

Input: Data A, C, and parameters λ, W, α, μ
Output: Optimal Matrix X with sparse diagonal

initialise $X = 0$, $D = 0$ and $B = 0$
repeat
  set $\hat{X} = X$ and $\hat{D} = D$
  repeat
    set $\overline{X} = \hat{X}$
    repeat
      compute optimal descent step size α.
      if *solving* (1) *or* (2) **then**
        $\overline{X} = \overline{X} - \alpha \left(A^\top \left(A\overline{X}A^\top - C \right) A + \lambda \left(\overline{X} - \hat{D} + B \right) \right)$
      end if
      if *solving* (3) **then**
        $\overline{X} = \overline{X} - \alpha \left(A^\top \left(A\overline{X}A^\top - C \right) A + \lambda \left(\overline{X} - \hat{D} + B \right) \right) \circ I$
      end if
    until *convergence towards* $\overline{X}^*$
    if *solving* (1) *or* (2) **then**
      $\hat{D} = \mathrm{shrink}_{\frac{\mu W}{\lambda}} \left(\overline{X}^* + B \right)$
    end if
    if *solving* (3) **then**
      $\hat{D} = \mathrm{shrink}_{\frac{\mu}{\lambda}} \left(\overline{X}^* + B \right)$
    end if
  until *convergence towards* $\hat{X}^*$ *and* $\hat{D}^*$
  set $X = \hat{X}^*$, $D = \hat{D}^*$ and $B = B - D + X$
until *convergence of* X, D *and* B
return X

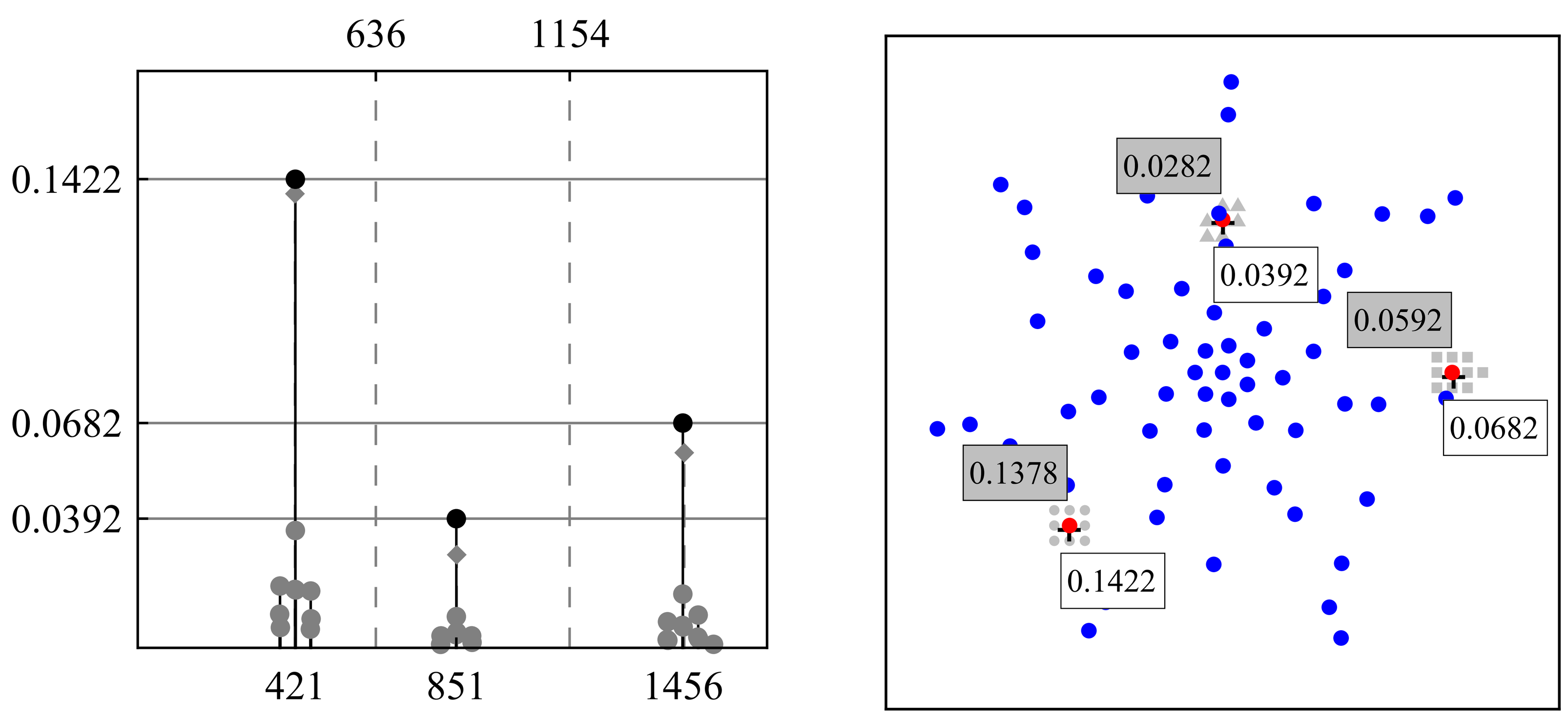
Post Processing (for noisy data)

Input: main diagonal of optimised matrix X
Output: clusters where centroid indicates source position and strength
begin
  remap the diagonal entries from X to actual positions in space.
  apply a k-means clustering to partition data.
  use the centroid position of each cluster as source position.
  sum up all source strengths from a cluster to obtain the source strength.
end
return *clustered data*

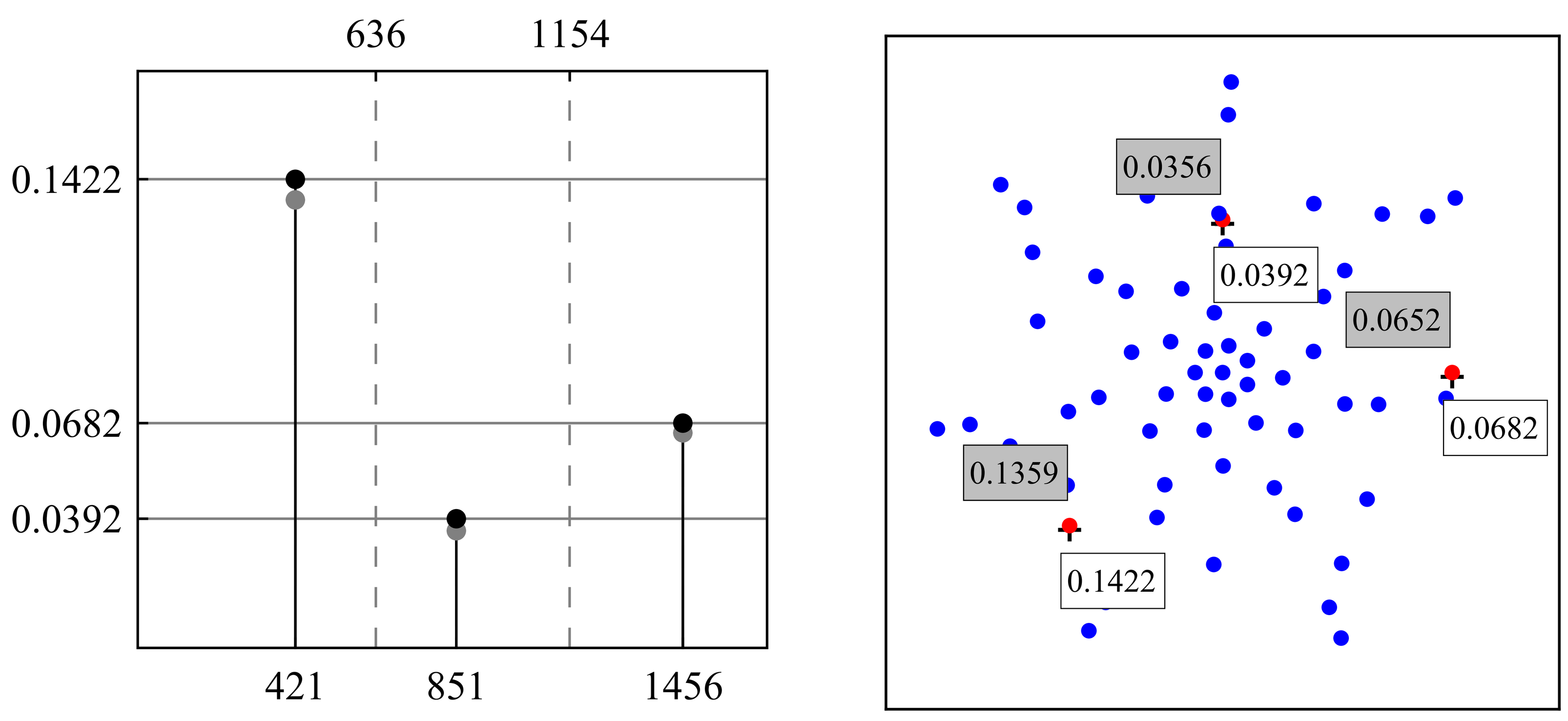
Results

- 3 sources, 1681 possible source locations on regular grid, 64 microphones
- data can be noisy and wrongly encoded (emitted from positions that shouldn't exist)
- left figure represents main diagonal of solution X (with clusters)
- right figure represents microphone array (in blue), correct signal strength (white labels), estimated positions (crosses) and clusters (grey shapes)

Model Eq. (2)



Model Eq. (3)



Conclusions

- almost perfect recovery with noise free data
- outperforms competing methods like CMF and Clean-SC on corrupt data
- post processing always yields desired number of sources
- fair convergence speed

- Laurent Hoeltgen, Michael Breuß, Gert Herold, Ennes Sarradj
Sparse ℓ_1 Regularisation of Matrix Valued Models for Acoustic Source Characterisation, Arxiv Report 1607.00171v1, 2016
- Gert Herold, Ennes Sarradj
Preliminary Benchmarking of Microphone Array Methods, Berlin Beamforming Conference, 2014